

How anthropology can contribute to mathematics education

Como a antropologia pode contribuir para a educação matemática

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Abstract

This paper starts from two statements based on a literature review. The first one concerns the learning process and states that learning is situated and socioculturally contextualized. Learning happens in the space of the background and the foreground of the learner in his or her particular environment of experience. This statement is based on the Vygotsky and the Cultural psychology approach (Cole, 1996) and on the work of Vithal & Skovsmose (1997).

The second statement concerns the deficient theory of the learning process (instead of the deficiency of the learner). Based on the international comparative research on mathematical skills we claim that the drop out of school of many groups of children (OECD, 2010) has to do with the insufficient learning system at school that fail to fit with the daily background knowledge of the children.

In the final part of the paper we will present three different ethnomathematical cases based on the educational practices that the authors developed in recent years.

Keywords: Ethnomathematics; Multimathemacy; Situated Learning; Navajo Indians; Turkish Immigrants, Urban Boundaries Project

Resumo

Este artigo começa a partir de duas afirmações com base em uma revisão de literatura. A primeira diz respeito ao processo de aprendizagem e afirma que a aprendizagem está situada e contextualizada socioculturalmente. A aprendizagem acontece no *background* e no *foreground* do aprendiz em seu ambiente particular de experiência. Esta afirmação baseia-se na abordagem de Vygotsky e a Psicologia Cultural (Cole, 1996) e no trabalho de Vithal e Skovsmose (1997).

A segunda afirmação diz respeito à teoria deficiente do processo de aprendizagem (em vez da deficiência do aluno). Com base em pesquisa comparativa internacional sobre as competências matemáticas reivindicamos que o abandono da escola por parte de muitos grupos de crianças (OECD, 2010) tem a ver com o sistema de aprendizagem insuficiente na escola que não se encaixa com o conhecimento do background diário das crianças.

Na parte final deste artigo apresentaremos três diferentes casos etnomatemáticos baseados nas práticas educativas que os autores desenvolveram nos últimos anos.

Palavras-chave: Etnomatemática; Multimatemacia; Aprendizagem Situada; Índios Navajos; Imigrantes Turcos; Projeto Fronteiras Urbanas.

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Introduction

The Vygotsky and the Cultural psychology approach (Cole, 1996) states that 'learning is situated, socioculturally contextualized'. Learning happens in the space of background/foreground of the learner in his or her particular environment of experience (Vithal, & Skovsmose, 1997). They have different languages, different cultural traditions, different social conditions which all entail differences in the shape and extent of the background-foreground of the learner. As Rosa & Orey (2011) state “An important change in mathematical instruction needs to take place in order to accommodate continuous and ongoing change in the demographics of students in mathematics classrooms” (p. 33). In a previous article on quilts, Rosa & Orey (2009) present an ethnomathematical project that helps students to better understand the concepts of symmetry and transformation, two central concepts of the geometrical part of the mathematics curriculum. The challenge for the mathematics teacher is to stimulate student's creativity and interest, based on the cultural and mathematical expressions of student's daily life and thus based on the student's background.

Maybe indigenous children are the example par excellence of a huge discrepancy between background and school culture. Mainstream school culture is characterized by a monolithic approach of academic mathematics (Bishop, 1988; Ernest, 1991).

Math learning implies an implicit (or sometimes explicit) understanding, categorizing and conceptualization of reality, e.g. set theory implies intrinsically a part-whole framing of reality which is awkward or counterintuitive to e.g. Cherokee, Athapaskan Native Americans, etc. These examples make it immediately clear that there exists a big discrepancy between what people know and what they have to learn; or to put it into the jargon of the learning theory, between the background knowledge of the learner and the school culture. This deficit within the learning process is not specific to indigenous children, the problem can be generalized to many groups of children.

We are certainly aware of the critical voices on international comparative empirical surveys that aim to assess mathematical literacy. Currently, two international surveys evaluate student performances: the Programme for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (formerly known as Third International Mathematics and Science Study – TIMSS). Both surveys are funded

by the International Association for the Evaluation of Educational Achievement (IEA). The criticisms are mainly related to the mathematics curricula politics. PISA and TIMSS are considered as ideological state apparatus at the macro-level that regulate behavior at the micro-level. National curricula of mathematics incorporate prescriptions for content, skills and competences in order to succeed at the international rankings. Therefore it is no longer the learner that has the primordial attention but the status of the country (Chronaki, 2011). Nevertheless the figures as they are presented at the recurrent reports provide us with some information we cannot deny. Even within the OECD new research is developed to inform and educate teachers for diversity in meeting the challenge (OECD, 2010). The new approach from the executive board is that diversity is an asset for educators and for society. Therefore efforts should be made to make the most of this rich resource of diversity. This approach contrasts with the conservative and nationalist view on diversity as a problem that needs to be solved or better to be avoided. From now, diversity is not a mere fact, it is valued as an asset for individuals and for societies in general.

Based on the figures of the international comparative research that aims to evaluate student mathematical performances we state that the tremendous dropout from math classes and the structural gap between good and bad performers is caused by disregarding the linguistic and socioculturally formatted background and foreground of the learners. When background knowledge of the learner differs substantially from the (implicit/intuitive) worldview of (school)mathematics, the drop out increases (OECD, 2010).

We propose to use anthropological study in the learning process (e.g. the classroom) to know and to map the child's background and foreground, and adapt the entry into mathematics courses accordingly. This paves the way for our option for multimathemacy –a concept we developed in Pinxten & François (2011)– as an alternative for a monolithic approach to mathematics.

The Challenge

Learning mathematics is a particular subset of learning. Hence, it is relevant to look at the learning theories which are available so far. Since we focus on learning in /of different cultural groups or populations in this paper, we went looking for an inclusive theory of learning. That is to say, one that is sensitive to context, culture and social differences. This means that we hold, as an a priori, that learning is a process that happens not only in the brain or even in the organism of a single individual. Rather, we see it as a process of change in the individual in interaction with the social, cultural and environmental contexts. Looking at learning in this way we were driven almost necessarily to the sociocultural learning theories of the Vygotsky branch, lately synthesized in the cultural psychology theory of Michael Cole. We will draw from this tradition on learning theories and disregard other ones (like S-R theory, Piaget's approach, etc.).

Learning is always 'situated' learning (Lave, 1988; Lave, & Wenger, 1991), and through manipulation of the contexts of learning in teaching settings, the process of learning can be influenced substantially.

Secondly, and consistent with the first choice, we focus on the characteristics of all parties in the process, when devising a curriculum and learning strategies. That is to say, the curriculum developers (in this case, the mathematicians trained in Academic Mathematics), the teachers (basically of the same background) and the pupils have their own mental setups when entering the learning process in a mathematics classroom.

We take this seriously and investigate what the input of all of them amounts to. When developing a curriculum and teaching procedures we will take all of these into account.

The basic reason why we feel obliged to go along this track (apart from mere ideological preferences for this or that societal model) is the fact that the drop out in the schooling is consistent and at the same time rather specific. In the overview report of the OECD two of the key factors are described as “-continuing disparities in scholastic achievement between first and second generation immigrant students and their native peers; -lower scholastic achievement and graduation rates for indigenous populations in countries with long history of migration” (OECD, 2010: 14). Data from the PISA 2003 and 2006 show that on average across all participating countries native students perform better in mathematics than first and second-generation immigrants (OECD, 2010: 24). This pattern is particularly troubling as it appears that native students perform better

than the second-generation immigrants who are born and raised at the same country. At the same time –and part of the explanation of second-generation immigrants’ situation– figures indicate that a student with low socioeconomic status is “twice as likely to be among the low achievers (OECD, 2010: 25). Concerning indigenous students the challenges identified across all countries (having indigenous populations that pre-date the arrival of European settlers viz. Australia, Canada, New Zealand and the United States) are the following: “difficulty in accessing and receiving the level of early childhood education and care recommended; lower levels of literacy and scholastic achievement; lower rates of graduation; proportionally higher representation in vocational education and training streams than their non-indigenous peers; and lower rates of participation in tertiary education in many of these countries (OECD, 2010: 26). This is some of the most important reasons why the executive board of the OECD states that educational systems have to become more effective and more equitable. These international research findings are confirmed by national research results, e.g. the Council of Australian Governments (COAG) states that Indigenous people are the most educationally disadvantaged group within Australia. Their educational outcomes are substantially lower than non-Indigenous students. For example, in 2006, year 12 completions for Indigenous Australians were 45,3% compared to 86,3% for non-Indigenous Australians (Howard, Cooke, Lowe, & Perry, 2011). If we know that Australians who not complete year 12 are less likely to have the same opportunities as those who do, we can speak in terms of inequality and violation of human rights.

We want to understand what is going on, and our proposal is that the learner’s perspective is not enough in the focus of math educational programs so far.

PISA research (OECD, 2005: 190) shows that good performance in mathematics education consistently link with high scores in reading and science knowledge. At the same time, low performance in school is uniform for a second group of the school population. The gap between both groups seems to consolidate or even widen, rather than narrow over the years. We interpret these results here as corroboration of our main thesis, namely that cultural and social differences between learners do count in education. That is to say, when pupils perform weak in the dominant language (which is more or less different from the home language) and in the dominant world view (for which the same can be said), then schooling which disregards in a general way these

differences -because of the “mainstream curriculum and ditto learning procedures”- will presumably yield larger gaps between subjects of the dominant social and cultural groups (i.e., middle class white groups) and others. We want to understand what is going on, and our proposal is that the learner’s perspective is not enough in the focus of educational programs so far. Here, we concentrate on math education only, starting from a general focus on learning in the first place.

The Learner’s Perspective. A General Sketch

The learner is not a mere receptive or passive party in our view. Hence, learning theories which ‘situate’ the learner and the learning process in contexts will carry our attention, and we will disregard the other ones. In a very general sense, we follow Cole’s (1996) synthesis in this respect.

In Vygotsky’s (1962) intriguing approach of almost a century ago, learning was first and foremost understood as a dialectical process between a learner and his or her environment. In other words, it was not identified merely with the processes inside the head of the learner or even at the edge of it. For example, in the very powerful stimulus- and response theory (behaviorism in its many versions) learning is studied as the result of the processing of (controllable) stimuli by means of the responses they trigger in an individual. Neither is it equated with a particular form of adaptational action on the part of the individual in his or her biological maturation cycle (as was the case in Piaget’s learning theory: Piaget, 1972). Vygotsky and his school broke away from these approaches and situated the learning processes plainly in the field of interaction between a learner and the physical, social (-historical) and cultural environment or set of contexts.

Such a focus has tremendous consequences for education. First of all, it entails that characteristics of both the learner and the environment matter in the curriculum and in the learning procedures. If the pupil is immature or otherwise unable to grasp the point of the learning process, then failure will ensue. But if the context is too poor, too far removed from anything understood or recognized by the pupil or in any other way

‘foreign’ to the pupil’s knowledge categories, then failure to learn will also be the result.

Secondly, it then becomes important to look for types of matching between the student’s mental setup and background knowledge and the challenges and possible inputs in the context. The latter could be hidden, openly offered, presented as triggers or otherwise entered in the interaction process with the learner. It is clear that learning procedures are in focus here.

Finally, evaluation of learning output stops being the assessment of the pupil’s responses only. It clearly and equally involves the assessment if the success or failure to induce learning by the contexts of the pupil.

The Learner’s Perspective. A Deeper Analysis

When we put the learner in the focus, it follows that we need to ‘flesh out’ the individual learner a bit more to go beyond the trivial. We side with a cognitive theory of the learner, claiming that some parts of the metaphorical ‘black box’ can be filled in a hypothetical, but nevertheless dependable way without losing scientific credibility.

In terms of mathematics education the best recent research in this area was done by Scandinavian colleagues in the research group of Skovsmose (Alrø, Ravn, & Valero, 2010). Skovsmose, the intellectual father of ‘Critical Mathematics Education’ (CME) situates mathematics education within a broad social and political context. Indeed to Skovsmose mathematics teaching and learning could aim at developing democratic competencies. This is why CME is concerned with mathematics education for all - independent of color, gender, class, CME is concerned with the practical application of mathematics –being an advanced technological application or an everyday use. It is also concerned with the democratic setting of a classroom situation, with the life in the classroom, and with the critical voice of pupils. A math class has to be a space of learning where ideas are presented and negotiated. Indeed to Skovsmose & Borba (2004) is CME concerned with the development of critical citizenship.

In this social and political embedded learning process, any learner is a subject within historical, social and cultural contexts, from which he or she brings into the learning

situation previously gathered concepts, problem solving strategies and learning procedures. These are summed up under the label of 'preschool knowledge'. Obviously, the contents of this category is primarily defined by the worlds of experience of the child: the peer groups, the family, the natural and sociocultural environment of the child. Hence, street children will differ in their preschool knowledge from Amazonian Indians, from city dwellers in Western Europe, from Aborigines and Torres Strait Islanders in Australia or from peasant children in rural China.

A further sophistication introduces the distinction between 'background knowledge' and 'foreground knowledge'. It was Vithal & Skovsmose (1997) who introduced the concept of *foreground* –besides the notion of background. Where the background means what children bring to the classroom, *foreground* is to be understood as 'the set of opportunities that the learner's social context makes accessible to the learner to perceive as his or her possibilities for the future' (Vithal, & Skovsmose, 1997: 147). Skovsmose (2005) emphasis also the political and cultural situation as an important aspect of the foreground since they provide the opportunities for the learner. It makes the political nature of the learning process explicit because it has to do with the student's possibilities in future life, not the objective possibilities as formulated by an external institution but the possibilities as the student perceives them.

The learner brings a 'background of knowledge' into the learning situation: he or she already has knowledge which is relevant for the issues or the problems presented in the school setting. E.g., each child has a mental map of the environment, which will allow to cover the distance between school and home in a rather efficient and safe way. At the same time, the child has a 'foreground of knowledge', which is the set of extensions of the knowledge that is acquired together with the skills to enable further learning. E.g., understanding of the school culture, management of problem solving techniques regardless of concrete contexts, and so on are examples of this. The child is actively learning in a classroom while making use of this 'mental setup'.

Learning and Culture in the Mathematics Classroom

Every couple of years the OECD assesses the quality of education throughout the world. These researches yield regular reports (the PISA reports), which give an overview of the success and failure ratio of children in and through schooling in math education. A recurring point in these reports is that (lower and higher) middle class pupils have a high success rate, whereas lower social classes and so-called ethnic groups are showing poor results (OECD, 2010).

Our hypothesis reads as follows: the common mathematics curriculum and teaching procedures start from the point of view that mathematicians (belonging to the group of so-called Academic Mathematics) define the school program in basic lines. The understanding seems to be that mathematics is what Academic Mathematics says it is. Obviously the immense sophistication of this type of knowledge and its proven worth in scientific and technological research feeds this status. The pupils then try to master that, equipped as they are with their particular sets of background of knowledge and foreground of knowledge. The systematic and clearly not decreasing failure in school mathematics of the groups mentioned, can hence only be blamed on these groups.

When we take the stand that learning is in actual fact always 'situated learning' and that the learner brings his or her background of knowledge and foreground of knowledge into the learning situation, we can draw the conclusion that the fact of disregarding the background of knowledge and the foreground of knowledge in the learners can explain why their performance is consistently poor. At the very least we can explain why more schooling does not automatically yield better results in mathematical education, especially for particular groups (as is shown in the PISA reports). But in order to do that, we have to take one more step.

Learners have background of knowledge and foreground of knowledge. But what about mathematicians, and their product of thought, i.e., Academic Mathematics? We propose the hypothesis that mathematical knowledge in Academic Mathematics has implicit categories, worldview notions, intuitions and the like in the language and the conceptual frame, which can be argued to be compatible with or translatable into that of a particular group of learners to a larger or smaller extent. Concretely, we suggest to investigate whether the middle class western subject's background of knowledge and foreground of knowledge is more easily translatable or more closely overlapping with the worldview

and categorization of Academic Mathematics than is the case with North American Indians, or lower class local groups and immigrant groups in Western Europe.

What we do by forwarding this hypothesis is not denying the tremendous worth of Academic Mathematics as a way of thinking and as a formidable tool for science and technology. Neither do we fall prey to a simplistic relativism, denying the high level of sophistication of this discipline. Instead, we claim that Academic Mathematics, like any human product, has its roots and that in learning Academic Mathematics these roots may show their relevance.

Let us develop this point somewhat. The structure of the Indo-European languages distinguishes between verb and noun forms. With this distinction corresponds a differentiation between things/states and operations/processes in the conceptualization of reality. Intuitively, mathematical thinking sophisticates these deep structural linguistic and cultural differentiations. Hence, the emphasis on geometric figures (with a thing-character) and their constitutive forms, on sets and their elements, on operations (of multiplication and so on) performed on entities (a number, a series, etc.). The point we want to make is that formal thinking elaborates the intuitive world view which is given in language and in folk knowledge (Atran, 1990). When investigating other cultural traditions we learn that Athapaskan and Cherokee languages, like Classic Chinese are 'verb languages'. That is to say, the noun category is inexistent or at least not substantial, corresponding to a view on reality as basically a world of events (Whitehead, 1906).

Again, regardless of the great achievements of Academic Mathematics, it is our conviction that it will be important to take these preschool differences into account for mathematics education. The dropout rates (cf. the PISA reports, OECD, 2010) might well be better understood in the light of these differences in preschool competences, to be found in the learner's background of knowledge. Following that line of reasoning, we must then conclude that it is likely that neglecting the background of knowledge of the child will yield lack of insights or more difficulties with the Academic Mathematics-inspired mathematics curriculum and learning procedures. Alternatively, we advocate to systematically involve the child's background of knowledge and foreground of knowledge in the educational process. By necessity this means that mathematics education (and hence also introduction to Academic Mathematics concepts and

theories) will have to take into account the different backgrounds of knowledge and foregrounds of knowledge of varying cultural traditions. In the following section we will elaborate on three most different examples of mathematics education programs with which the authors have been involved.

Closing the Gap. Mathematics Education Programs

In this final section we offer some suggestions about the practical use of ethnomathematics both inside and outside the classroom, taking as a starting point examples of field research the authors are involved in.

The first example on Navajo Indians (in the USA) will explain how a specific aspect of language and formal thinking can be included in a mathematics curriculum to introduce the Navajo child into the basic elements and notions of Academic Mathematics.

A second example among the Turkish ethnic minority (in Belgium) shows how multimathemacy can enter the classroom by using a didactical model of interaction and working with heterogeneous groups (migrants and autochthonous) of children.

In a final example we illustrate an 'out of school' project with two immigrant communities situated in the outskirts of Lisbon (Portugal).

Navajo Indians, USA

In Navajo Indian culture, the verb 'to go' is conjugated in no less than 300.000 ways. On the other hand, no counterpart for the verb 'to be' exists. This contrasts sharply with the Indo-European languages in which Academic Mathematics emerged and is proliferating. Actions in Navajo begin, stop, change or transform. The cosmos can be understood as a universe of events, rather than a universe of things. In such a view no part-whole logic of 'beings' or 'objects' and their elements obtains (Pinxten, van Dooren, & Harvey, 1983). In Academic Mathematics, on the other hand, the very basis for formal reasoning is a part-whole logic: the world of experience is split in parts. For example, in geometry a line is defined as a set of points, or a plane is said to be a set of lines.

The central place held by verbs ‘to go’ in Navajo yield a perception and conceptualization of form or another aspect of space as determined or defined by movement. E.g., an edge is not a line which forms the limit of a surface or a volume. Rather it is an obstacle in the action, or a happening which changes the movement. For example, when progressing uniformly over a surface of a table, the movement is suddenly stopped or altered in a fall: that is what is called an edge.

A former project was to work this sort of data into a curriculum for elementary geometry for Navajo children, starting from their language and world view. The background of knowledge of the reservation child includes environmental knowledge, often based on herding sheep (alone or with an adult) in the canyon lands (Pinxten, van Dooren, & Soberon, 1987; Pinxten, & François, 2011). The child pictures the landscape by means of ‘significant’ rocks (i.e., with particular shapes), the adjacency of dips and water sources, the movements of the sun throughout the day, eventual greenery at different places, the entry of the hogan one finds along the path (i.e., the door of the hogan or dwelling place will always point to the east), the changes of color of the air throughout the day (going from white in the morning, over blue and yellow to black), and so on. All of these elements co-define distance in the Navajo view.

If we want to introduce the child into academic mathematical notions, we propose to start from this background of knowledge of the children and identify their foreground of knowledge on the basis of it. That is to say, we start from the preschool and outside of school knowledge, make the concepts and intuitions explicit and label them in the native language. E.g., a line is conceptualized as the result of the uninterrupted movement of the child through a landscape. It corresponds more to a path and is closely tied to the moving body. Or it can be understood as the result of two movements in a plane, one rock plateau spreading towards a different moving progression of what we call a river. Where the two movements meet we speak about a line. When looking at the world of experience of the Navajo child, one notes that ‘a river’ is not a continuous thing with minor changes in extension. Rather it is mostly a dry bedding and sometimes a genuine torrent in these areas. In a next step we can then go on and label ‘line’ in Navajo as a particular movement. Next, we explore the characteristics of the line explicitly and only then move to the lemmas and problems defined in the Euclidean geometry system. In this way, multimathemacy is shaped in the classroom practice.

Turkish Immigrants in Flanders, Belgium

Turkish migration to Belgium was actively encouraged by Belgium from 1964 when Belgium signed a bilateral agreement with Turkey to meet man power needs in an era of rapid economic expansion. Turkish people -mostly unskilled laborers- were welcomed as guest workers and were mainly received by Belgian coalmines. The Turkish ethnic minority in Belgium originates predominantly from rural areas in Turkey. Most of them originate from the region of Emirdağ (west-central Turkey) although there are also many Turks from Sivas (east-central Turkey) and from Kars (east Turkey). Hence, their background of knowledge is to a considerable degree pre-Newtonian and more broadly speaking not adapted to Flemish school culture, although their distance to European subjects is of course much smaller than that of Navajo children. During our observations (Pinxten, & François, 2007) we particularly found two differences in the daily world of experience: the tasks in mathematics education are often middle class Eurocentric (e.g. about Western modern city life instead of about Turkish rural country life), which for Turkish pupils causes misunderstandings. Moreover, the Turkish language is sometimes a source of difficulties in a Flemish school context: plurals are not shown in nouns, but an appellative usually replaces numerals and the plural. Moreover, in Turkish, there is no such thing as a prefix, which leads to difficulties with 'self-evident' spatial references like above, under, beside, etcetera. These few remarks give an idea of the problem confronting us. We propose the following curriculum project in Flanders to overcome the difficulties.

Children in mixed class groups (migrants and autochthonous) are asked to discuss their neighborhood. Concepts like centre, border, near and far off, etcetera are described in both Turkish and Dutch. The children are encouraged to work together across cultural borders. Translation problems that crop up in the process are largely left to the children themselves. The building of scale models and the development of graphic presentations of the neighborhood allows them to get a grip on the differences and commonalities in meaning and speech about abstract notions such as distance, size, proportion, geometrical shapes –after all, we are working in a highly geometricised urban environment. During this phase it is important to explicitly refer to and work on the linguistic differences mentioned: in the practice of multicultural schools we observed

that Turkish children started interfering in the classroom with their own ‘code’. They translated and illustrated their insights in Turkish for the classmates who lost track when the teacher progressed with his or her ‘western’ presuppositions. These children saw the growing gap of misunderstanding and started interjecting what they saw as bridges for an insightful progression. They did this in Turkish, with common background of knowledge for the Turkish children. Our suggestion, to enrich the mathematics curriculum in a way of integrating multimathemacy, is to honor this supplementary code of the children and integrate it in the teaching practice. That is to say, new to the first code (that of the Flemish mathematics curriculum, dealing with the European notions and examples), and the second code (that of the teacher as an authority in the classroom), we suggest that the teacher recognizes and integrates this third code of learning (emanating purely from the background of knowledge of the child). Integrating it in the teaching process means that the Turkish notions and terms should be allowed and worked upon explicitly: what is the difference in meaning between the curricular concept and terminology, and that of the Turkish rural country tradition? Where is the link between both, and what are the possible bridges in understanding between both? Again, one should be carefully not to depreciate or reduce the input by the pupils, but rather take their concepts and insights as a supplementary starting point for the teaching process. This implies that also concepts and problem solving procedures from the background of knowledge of the children should be taken seriously in the classroom. In this way, again, the foreground of knowledge of the children is actively built up in the process of teaching. This then is another instance of realizing multimathemacy. Even if one appreciates the differences between the contents and even the extent in the two examples, the general notion of a multimathemacy approach will shape up.

In a final example we will illustrate an out of school project.

Urban Boundaries Project, Lisboa, Portugal

Recently we started an international and interdisciplinary academic project named Urban Boundaries; a project which is in full progress. In this section we will present the context of this project from a geographical, historical, social and political point of view. Further one we present the people who are involved in the project and we discuss the

importance of the voices —desires/needs/possibilities— of the actors enrolled in the out of school project. The theoretical framework of this out of school project is based on the ‘Curriculum Trivium’ as proposed by Ubiratan D’Ambrosio (1990) and on the recent philosophical critique on multiculturalism as developed by the Slovenian philosopher Slavoj Žižek. Slavoj Žižek (1989) offers us tools to understand the contradictions of current society where, despite the increasing abundance and sophistication of resources available, communities such as the ones we work with continue to lack the basic conditions for a live with dignity.

In line with the theoretical framework, our methodological approach is informed by Critical Ethnography.

The Urban Boundaries is an academic project supported by Fundação para a Ciência e a Tecnologia of Portugal and Lisbon University. It is constituted by a group of researchers from different backgrounds (among others, architects, biologists, philosophers, physicists, teachers, and mathematics educators) that decided to join efforts and built a project together with two ‘out of school’ communities, one situated in an agricultural context and the other in a fishing community, both placed in the outskirts of Lisbon—a place called Costa de Caparica. These communities are constituted by immigrant populations from other Portuguese-speaking countries, Gipsies, and Portuguese migrants. They have been experiencing throughout half a century diverse problems of inclusion, particularly concerning schooling and the need to have their voices heard. Through the development of a critical alphabetization, mathematical literacy, a multiple cartography and life-history portfolios we seek to address the educational needs in terms of foreground of knowledge of these populations *in situ*. With the notion of *in situ* we refer to the background of knowledge of these people, that is, people in the midst of their everyday lives where survival with dignity is often the first and foremost important daily struggle. Therefore, it is the everyday problems felt by these two communities that guide the organization of parameters that support an intercultural education curriculum based on the sociocultural and economic reality of these communities.

Especially concerning mathematical literacy and multimathemacy, we wish to engage the participants in a discussion about the importance of mathematics outside a frame of schooling, namely the role this science can have in the struggle for survival that both

communities experience in a daily basis. Therefore we introduce D'Ambrosio's ethnomathematical program as an all-encompassing societal program based on the idea that there are several ways, techniques, skills (tics) to explain, understand, deal with and live with (mathema) distinct natural and socioeconomic realities (ethnos). D'Ambrosio (1990) was the first, from the late 1980s, to propose a research program for ethnomathematics, based on the following analysis of the term.

I call mathema the actions of explaining and understanding in order to survive. Throughout all our own life histories and throughout the history of mankind, techn'es (or tics) of mathema have been developed in very different and diversified cultural environments, i.e. in the diverse ethnos. So, in order to satisfy the drives towards survival and transcendence, human beings have developed and continue to develop, in every new experience and in diverse cultural environments, their ethno-mathema-tics (p. 369).

Following this, the research interests of the newly founded discipline ethnomathematics pertain to the development, transmission and distribution of mathematical knowledge as dynamic processes embedded in their sociocultural context.

Against this background, ethnomathematics appears not so much as the study of 'different mathematics', but as a way to deal with different forms of 'knowing' (Mesquita, Restivo, & D'Ambrosio, 2011).

In the Urban Boundaries Project (as is the case in the Navajo Indians and the Turkish immigrants' project) ethnomathematics is not to be confused with a subfield of mathematics education, designed to improve school mathematics, but as a political space where new forms of emancipation can be thought and practiced.

Final Considerations

In this article we continued the research project of cognitive anthropology (Pinxten et al., 1983; Atran, 1990) within the research field of mathematics education.

The theoretical framework of our research departs from Vygotsky's sociocultural learning theory and the Cultural psychology together with the results of Vithal & Skovsmose's (1990) theory on Critical Mathematics Education. From this theoretical framework we can take the stand that learning is always 'situated learning' and this also applies to mathematics education. Learners always bring

their background knowledge into the learning process. In line with the broad concept of ethnomathematics as developed by D'Ambrosio, we proposed at the same time that even Academic Mathematics is culturally embedded. Academic Mathematics has its own (implicit) categories, worldviews and applications. The challenging question remains how to close the gap between on the one hand the diverse community of the learners and on the other hand the community of the learning institution (school culture, curriculum, teacher, ...). In order for teachers to deal with the diversity of backgrounds of knowledge, they should have knowledge of these cultural backgrounds, traditions, languages, practices and mathematical practices the learner (can) bring into the learning context. Teachers should have an anthropological perspective on the learning process, on the school culture and on the diverse cultures the learners bring to school. That is why we propose to use anthropological study in the learning process in general.

As an alternative to the monolithic approach to mathematics we can now pave the way for our option for multimathemacy. Multimathemacy is an educational perspective that invites the teaching of different cultural insights on counting, proportional thinking, mapping or spatial organization in pre-school and out of school knowledges and this view offers bridges between academic mathematics and cultural knowledge traditions for schooling.

In the empirical part of the paper we presented three different cases –based on our own educational practices– to illustrate the ways in which the theoretical concept of multimathemacy can be put into practice. Based on the theoretical framework and on the empirical practices in multimathemacy we can state (i) that world view counts in mathematics education and (ii) that the particular procedures of learning count. We propose an anthropological approach in the mathematics learning process to compensate the deficiencies in school.

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